Enrollment No: _______________________ **Exam Seat No:**_______________________

C.U.SHAH UNIVERSITY
Summer Examination-2018

Subject Name : Complex Analysis

Subject Code : 4SC05CAC1 **Branch:B.Sc.(Mathematics)**

Semester : 5 **Date : 21/03/2018** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:
1. (1) Use of Programmable calculator & any other electronic instrument is prohibited.
2. (2) Instructions written on main answer book are strictly to be obeyed.
3. (3) Draw neat diagrams and figures (if necessary) at right places.
4. (4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $f(z) = u + iv$ in polar form is analytic then $\frac{\partial u}{\partial r}$ is (01)

a) $\frac{\partial v}{\partial \theta}$ b) $r\frac{\partial v}{\partial \theta}$ C) $\frac{1}{r}\frac{\partial v}{\partial \theta}$ d) $-\frac{\partial v}{\partial \theta}$

b) A function u is said to be harmonic if and only if (01)

a) $u_{xx} + u_{yy} = 0$

b) $u_{xy} + u_{yx} = 0$

c) $u_x + u_y = 0$

d) ${u_x}^2 + {u_y}^2 = 0$

c) The function $f(z) = |z|$ is non constant (01)

a) Analytic function b) Nowhere analytic function

c) Non analytic function d) Entire function

d) If $e^{ax}\cos y$ is harmonic then a is (01)

a) i b) 0 c) -1 d) 2

e) The region $|z| > 1$ represent (01)

a) Exterior of unit disk b) Open unit disk

c) Closed unit disk d) None of these

f) Transformation $W = \frac{1}{z}$ is known as (01)

a) Inversion b) Translation c) Rotation d) None

g) The fixed points of the transformation $W = z^2$ are (01)

a) 0,1 b) 0,-1 c) -1,1 d) –i,i

h) The bilinear transformation that maps the points $0, i, \infty$ respectively into $0, 1, \infty$ is $W =$ ___. (01)

a) $1/z$ b) $-z$ c) $-iz$ d) iz

i) State Liouville's theorem. (01)

j) Write Cauchy-Reimann equation. (01)

k) If $f(z) = x + ay + i(bx + cy)$ is analytic then find a, b, c. (02)

l) Find the harmonic conjugate of $2x - x^3 + 3xy^2$. (02)

Attempt any four questions from Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	a)	State and prove Cauchy Riemann equation.	**(07)**
	b)	Show that $(z) = \begin{cases} \sqrt{xy} & ; z \neq 0 \\ 0 & ;\ z = 0 \end{cases}$, satisfy Cauchy-Riemann equation but not differentiable at 0.	**(07)**
Q-3		**Attempt all questions**	**(14)**
	a)	State and prove Cauchy integral formula.	**(06)**
	b)	Evaluate $\int_c (x - y + ix^2)dz$,where c along real axis from $z = 0$ to $z = 1$ and then the line joining $z = 1$ to $z = 1 + i$.	**(05)**
	c)	Prove that $f(z) = \bar{z}$ is no where differentiable.	**(03)**
Q-4		**Attempt all questions**	**(14)**
	a)	State and prove Liouville's theorem.	**(05)**
	b)	Find analytic function such that $Re\ (f') = 3x^2 + 4y - 3y^2$ and $f(1 + i) = 0$.	**(05)**
	c)	Find the value of integral $\int_c \frac{dz}{z^3(z+4)}$ where $c: \lvert z \rvert = 2$.	**(04)**
Q-5		**Attempt all questions**	**(14)**
	a)	State and prove Morera's theorem..	**(05)**
	b)	Show that $u(x, y) = e^y(\cos x + \sin x)$ is harmonic. Find harmonic conjugate of $u(x, y)$ and $f(z)$.	**(05)**
	c)	Evaluate $\int_c (z - z^2)dz$, where c is the upper half of the circle$\lvert z - 2 \rvert = 3$.	**(04)**
Q-6		**Attempt all questions**	**(14)**
	a)	Prove C-R equation in polar form.	**(05)**
	b)	Function $u = \log r$. If u satisfy $r^2u_{rr} + ru_r + u_{\theta\theta} = 0$ then u is called harmonic function find its conjugate v.	**(05)**
	c)	Analytic function of constant modulus is also constant in its domain D.	**(04)**
Q-7		**Attempt all questions**	**(14)**
	a)	State and prove Cauchy's theorem.	**(06)**
	b)	Find $\int_{1-i}^{2+3i} (z^2 + z)dz$.	**(04)**
	c)	State and prove ML inequality.	**(04)**
Q-8		**Attempt all questions**	**(14)**
	a)	Find image of $\lvert z + 1 \rvert = 1$ under the transformation $W = \frac{1}{z}$.	**(05)**
	b)	Find mobious transformation that maps the points $z_1 = -1, z_2 = 0, z_3 = 1$ on to $w_1 = -1, w_2 = -i, w_3 = 1$ respectively.	**(05)**
	c)	Prove that the transformation $(w + 1)^2 = \frac{4}{z}$ transform the unit circle of $w -$ plane into the parabola of $z -$plane.	**(04)**